

# Prediction of Ultimate Load Capacity of Tapered Members Using Artificial Neural Network

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**Abstract** Members with non-prismatic geometry are commonly used in building or bridge structures. One of the important checks in the process of design of this type of members is the control of their buckling load capacity. In this paper, the backpropagation feed-forward artificial neural networks are implemented to predict the ultimate load capacity of tapered beam-columns. For this purpose, 1260 models with different cross-section geometry, boundary condition and initial geometry imperfection are created and used for training purposes of neural network. Result shows that over 95 percent of predictions were between 0.9 to 1.1 times the actual values.

**Index Terms** Ultimate Load Capacity; Tapered beam-columns; Artificial Neural Networks.

## 1. INTRODUCTION

Utilization of non-prismatic or tapered members as beam, column, or beam-column member in a structure causes a smooth stress distribution along the member and reduces the consumption of materials [1]. These are the two most important reasons that lead designers to use this type of member's configuration.

Literature of last couple of decades includes several researches in which the structural behavior of this type of elements has been examined. Performed researches include the topic of torsional buckling, flexural buckling, and free vibration analysis of tapered members [2-5].

When tapered member acts as a beam-column, then, considering the effects of axial load on stress distribution along the beam becomes important. In this situation, the buckling load of a tapered beam-column member becomes as one of the important design checks.

There are some researches in literature that has studied the concept of buckling and instability of tapered beam-columns [6-11]. The implemented methods of studies in these articles could be classified into two major groups:

- Numerical modeling, and
- Close-form solutions

Numerical modeling methods, such as the finite element modeling, are one of the analysis methods in which the tapered member is divided into a number of uniform elements and the ultimate load is calculated by using this step model. The buckling load can be achieved by finite element analysis, although, it is still time consuming and lowly efficient especially when tremendous number of members exist in the considered system [6-8].

Another studied method is to find a close-form solution for the governing differential equation [1, 4, 9-11]. Because of existence of many boundary conditions and nonlinearity in this problem, it is tough to find a theoretical solution. Moreover, resultant theoretical solution is not that much handy to be used.

According to the best knowledge of authors, proposed methods are based on either numerical modeling or closed-form solutions which both are not efficient. Hence, it seems that it is worthy to find a reliable and fast method for prediction of buckling load of non-prismatic members.

Artificial neural network as a well-known method in the field of artificial intelligence is an analytical model which is a mimic of human neural system and its input/output association. This is a powerful pattern recognizer and is particularly suitable for solving complex problems. There different types of ANN in the literature [12, 13]. Each model is designed for a specific purpose. Back propagation feed-forward artificial neural network is one the most common type of ANN which is a cost effective and less time consuming mean of predicting the results of complicated functions. MATLAB has a complete toolbox for ANN. This toolbox has applicability in a variety of research fields [14, 15], especially in Civil Engineering [16, 17].

When mathematical explicit formula of a function is unavailable or tough to be achieved, but, a finite number of input/output functional relations of considered function are known, artificial neural networks are capable of modeling and mimicking these relations and establish these relationships to predict the solution for a new input vector. This is achieved by training the artificial neural network on a known dataset.

A backpropagation feedforward artificial neural network contains three layers; input layer, hidden layer(s), and output layer. The process of training includes weight adjustment and statistical optimization of predefined variables inside these layers. After achieving a trained neural network, its performance will be evaluated based on testing data.

Goal of this paper is to use the artificial neural networks to predict the ultimate load bearing capacity of tapered beam-column. For this purpose, different models of non-prismatic members are created and their ultimate loads are calculated. Then, a twin member, which has the same configuration as original member, but, its cross-section is prismatic is created for each model and its ultimate load is calculated. Finally, the ratio of ultimate load of original member over its twin is calculated. The ANN is trained based on these data. Resultant trained artificial neural network can be used to predict the ultimate load of any new tapered beam-column.

## 2. PROCEDURE OF ANALYSIS

Zeinali et al. [4] presented the general solution of the arguments of the stiffness matrix of a tapered beam-column when both axial and lateral load exist. By using Chebyshev polynomials method the closed-form solution for this model has been found in reference [4].

Utilizing this method, a second order analysis of tapered beam column is performed to achieve the relationship between the axial load and lateral deflection, i.e., P-Delta curve. In this model, an axial load is applied on one end of the member and an initial geometrical imperfection is considered for whole member. Because of existence of this initial imperfection, axial load will create a secondary moment along the member. For that reason, if a second order analysis is performed on this member, then, increasing the axial load will increase the mid-span deflection.

In this research, the magnitude of axial load is increased incrementally and the corresponding lateral movement of the mid-span of the beam is monitored. Eventually, by plotting these load values and corresponding deflections, the so-called P-Delta curve could be achieved.

As it can be seen from this plot, resultant curve smoothly converges to a specific value. This value is called the buckling or ultimate load capacity of the member. By this method, the buckling load of all models has been calculated. A typical form of resultant P-Delta curves is illustrated in Fig. 1. In this figure, the limit of convergence of curve is presented

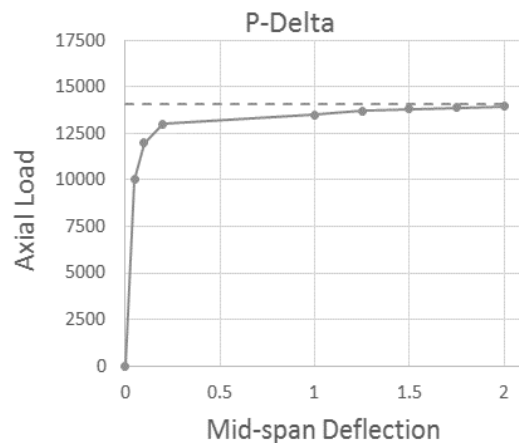


Fig. 1. A typical P-Delta curve.

by dashed line. This curve converges to the lowest value that is calculated by Eigen value problem.

Then, another model with the same boundary condition and initial imperfection is created. The only difference in this new model is that member is not tapered and has a uniform cross-section. The moment of inertia of this new member is the average of all cross-sections in original tapered beam.

The theoretical solution of buckling load of a member with uniform cross-section with different boundary conditions is already studied. The goal of this paper is to predict the ratio of buckling load of tapered beam over the buckling load of its twin beam-column which has uniform cross-section.

In this study, a member with I-shape cross-section is considered. The member is divided into four segments. End of each segment is considered as a station. This shape of cross section at each station is known, although, just the depth of I-shape cross-section varies along each segment. The moment of inertia at the first cross-section is considered equal to  $I_1$  and the values for other stations are defined based on first station. Considered values for second moment of inertia for stations 2 to 5 are presented in Table 1.

Member has one span. So, geometrical boundary conditions are defined at the end of the beam. Each end has three degrees of freedom. Although, left end is always assumed to be fixed in x direction and right end translation in x-direction is assumed to be always free. By this assumption, Boundary conditions that are tabulated in Table 2, are the remaining possible ones and all are considered in this study.

Another variable along considered models is the magnitude of initial geometrical imperfection. Higher value for this imperfection tremendously reduces the ultimate load capacity or buckling load of member. In this study, a sine wave with maximum deflection at the center of the beam was considered. For the maximum deflection,  $\delta_0$ , 6 different values are

considered. Considering all variables, totally 1260 instances of input and output vector would be made.

Table 1. Assumed second moment of area.

Case	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>	I <sub>5</sub>
1	0.875I <sub>1</sub>	0.750I <sub>1</sub>	0.625I <sub>1</sub>	0.50I <sub>1</sub>
2	0.750I <sub>1</sub>	0.500I <sub>1</sub>	0.750I <sub>1</sub>	0.50I <sub>1</sub>
3	0.500I <sub>1</sub>	0.500I <sub>1</sub>	0.500I <sub>1</sub>	1.00I <sub>1</sub>
4	1.250I <sub>1</sub>	1.500I <sub>1</sub>	1.750I <sub>1</sub>	2.00I <sub>1</sub>
5	0.500I <sub>1</sub>	0.500I <sub>1</sub>	0.500I <sub>1</sub>	0.50I <sub>1</sub>

Table 2. Assumed boundary conditions.

Case	δ <sub>L</sub>	θ <sub>L</sub>	δ <sub>R</sub>	θ <sub>R</sub>
1	1	0	1	0
2	1	0	1	1
3	1	0	0	1
4	1	1	1	0
5	1	1	0	1
6	1	1	1	1

Input vector is a vector with ten components. The definition of each component is as below:

$$\text{Input} = \left( \frac{I_1}{L}, \frac{I_2}{I_1}, \frac{I_3}{I_1}, \frac{I_4}{I_1}, \frac{I_5}{I_1}, \delta_L, \theta_L, \delta_R, \theta_R, \frac{\delta_0}{L} \right) \quad (1)$$

In which, *L* is the span length, δ<sub>0</sub> is the initial geometry imperfection at the center. Other values can be achieved based on values inside Tables 1 and 2.

The corresponding output to each input vector is the ratio of ultimate load of tapered beam-column over the ultimate load of its twin beam.

$$\text{Output} = \left( \frac{N_{cr \text{ tapered}}}{N_{cr \text{ twin}}} \right) \quad (2)$$

A standard feed-forward backpropagation neural network with 10 neurons in the first hidden layer and one neuron in the second hidden layer is used. Using stratified cross-validation technique, 70 percent, 15 percent and 15 percent of dataset is dedicated to training, validation and testing stages respectively. An error function in the form of the sum of the squares of the errors between the calculated outputs from the actual value of targets is defined and iteratively minimized.

### 3. Results

Figure 2 presents the structure of considered artificial neural network. It can be seen that considered transfer function of hidden layer is a tangent hyperbolic function while the transfer function is a linear function in output layer. There 10 neurons in input layer, 10 neurons in hidden layer, and one neuron at the output

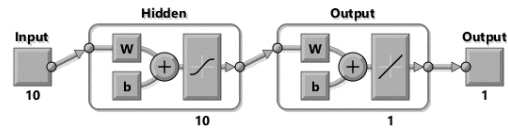


Fig. 2. Considered ANN architecture.

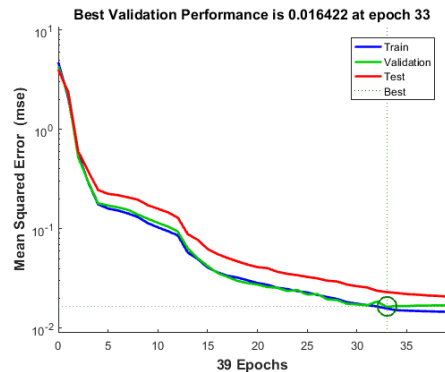


Fig. 3. Mean square error (MSE).

layer. By this configuration, there will be below number of unknown coefficients in this artificial neural network:

$$\text{No. of unknowns} = 10 \times 10 + 10 \times 1 + 10 + 1 = \boxed{121} \quad (3)$$

So, there will be 121 number of unknown in this ANN which is a logical selection comparing to 1260 number of training and testing instances. This relation between number of unknowns and number of training instances guarantees that no over-fitting problem will be occurred.

Figure 3 shows a line graph of mean square error (MSE) for training versus the number of epochs. The training and validation process mostly controlled based on these results. It can be seen that the best validation performance happened at epoch 33 means that process is terminated based on the values resulted from this epoch and the testing process starts.

Figure 4 shows the correlation between the outputs and actual targets from dataset. For both curves a regression value higher than 0.99 can be observed. It means that the resultant ANN will show a very good performance.

Presented plot are generated by built-in functions in MATLAB toolbox. Besides of these plots, authors suggest another plot which illustrated in Fig. 5. Figure 5 presents the ratio of values predicted by ANN over the real values from training and testing dataset for all existence samples. Theoretically, this ratio should be

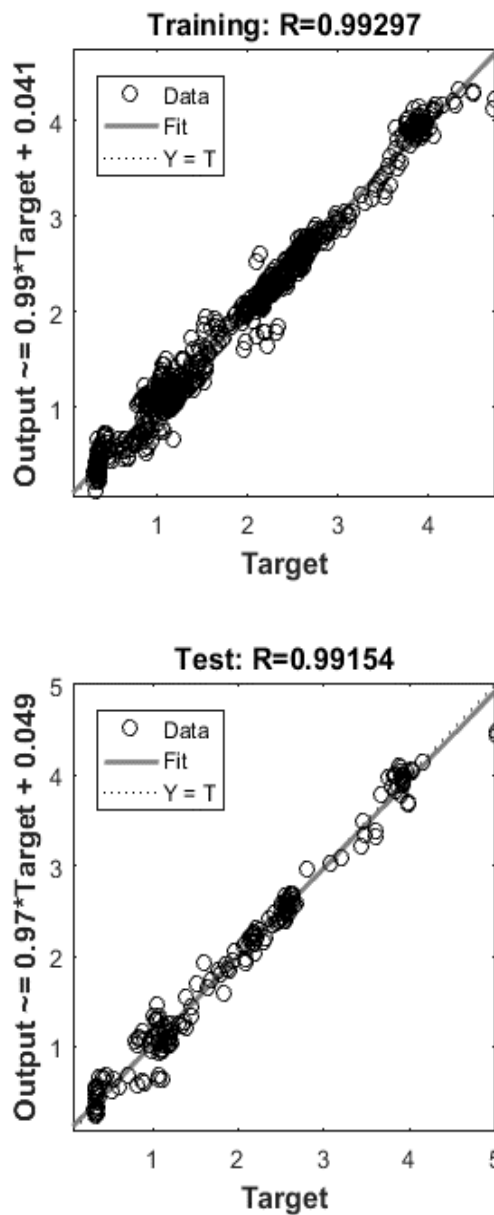


Fig. 4. Regression plot for training and testing steps.

equal to one, but, in practice, values between 0.5 and 1.5 are very good results.

It can be seen that 95% of the resultant values are between 0.9 and 1.1. This means that the trained network is capable to give reasonable prediction of ultimate load capacity.

#### 4. Conclusions

Members with non-prismatic geometry are commonly used in building or bridge structures. One of the important checks in the process of design of this type of members is the control of their buckling load

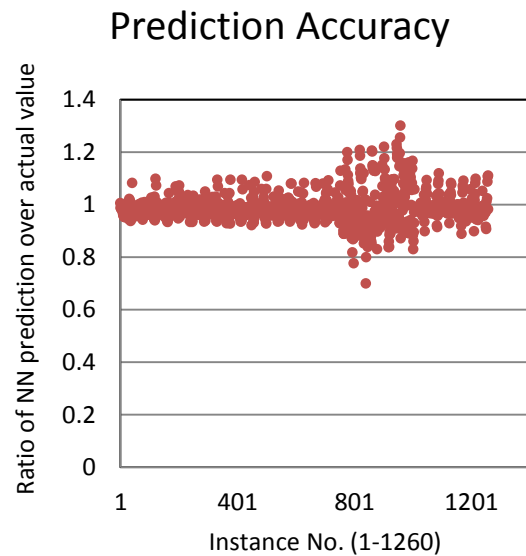


Fig. 5. ANN prediction accuracy.

capacity. In this paper, the backpropagation feed-forward artificial neural networks are implemented to predict the ultimate load capacity of tapered beam-columns. For this purpose, 1260 models with different cross-section geometry, boundary condition and initial geometry imperfection are created and used for training purposes of neural network.

Below conclusions can be drawn based on the results achieved in this paper. The ultimate load bearing capacity of a tapered beam-column can be achieved by finite element method or a close form solution. In this paper, another approach has been implemented. In the presented method, a series of models with different geometry, boundary conditions, and initial imperfections configurations is constructed. The ultimate load capacity of this set is calculated.

Also, the ultimate load capacity of twin system of each considered model is calculated too. Twin system has the same boundary condition and initial geometry imperfection configuration except as it cross-section is a uniform cross-section. Then, the ratio of ultimate load capacity of original model over the twin structure will be calculated. An artificial neural network is trained based on these inputs and outputs. It has been observed that resultant network is able to predict the actual targets reasonably acceptable. Using resultant network, the ultimate load capacity of a new member can be almost accurately predicted.

Result shows that over 95 percent of predictions were between 0.9 to 1.1 times the actual values.

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